< 1.5. Aerodynamic forces and moments >

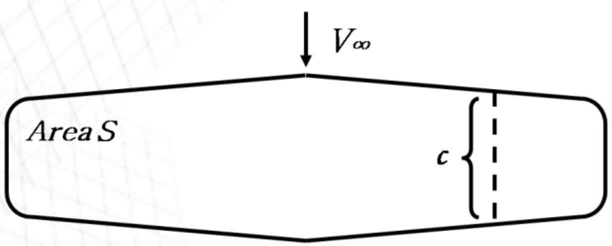
❖ Dimensionless coefficients

- The forces and moment depend on a large number of geometric and flow parameters. It is often advantageous to work with dimensionless forces and moment, for which most of these parameter dependencies are scaled out. For this purpose we define the following reference parameters:
  - Area : S
  - Length : l
  - Dynamic pressure : $q_\infty = \frac{1}{2}\rho V^2$

< 1.5. Aerodynamic forces and moments >

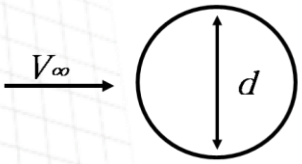
❖ Dimensionless coefficients

- The choice for S and I are arbitrary, and depend on the type of body involved. For aircraft, traditional choices are the wing area for *S*, and the wing chord or wing span for *l*.



(*a*)

S = plan-form area
$l = c$ = mean chord length



(*b*)

S = cross-sectional area
$l = d$ = diameter

< 1.5. Aerodynamic forces and moments >

❖ Dimensionless coefficients

- These local coefficients are defined for each spanwise location on a wing, and may vary across the span. In contrast, C_L, C_D, C_M are single numbers which apply to the whole wing.

< 1.5. Aerodynamic forces and moments >

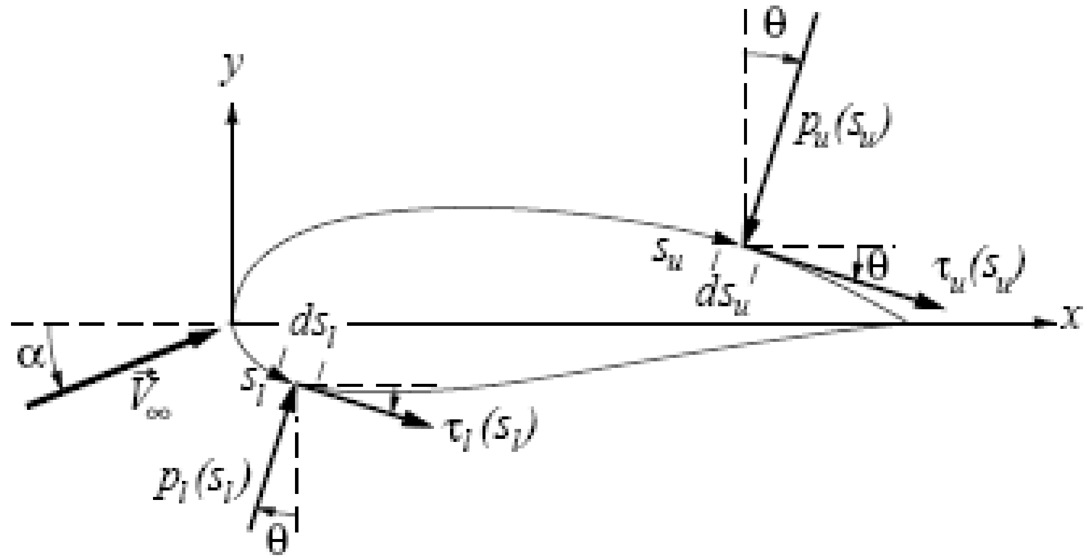
❖ Dimensionless coefficients

- From the geometry, we have

$$dx = ds\cos\theta$$

$$dy = -\left(ds\sin\theta\right)$$

$$S = c(1)$$



< 1.5. Aerodynamic forces and moments >

❖ Dimensionless coefficients

- Using above relations, we obtain the following integral forms for the force and moment coefficients :

$$c_n = \frac{1}{c}\left[\int_0^c \left(C_{p,l} - C_{p,u}\right)dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right)dx\right]$$

$$c_a = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,l}\frac{dy_l}{dx}\right)dx + \int_0^c \left(c_{f,u} + c_{f,l}\right)dx\right]$$

$$c_{m_{LE}} = \frac{1}{c^2}\left[\int_0^c \left(C_{p,u} - C_{p,l}\right)x\,dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right)x\,dx \right.$$

$$\left. + \int_0^c \left(C_{p,u}\frac{dy_u}{dx} + c_{f,u}\right)y_u\,dx + \int_0^c \left(-C_{p,l}\frac{dy_l}{dx} + c_{f,l}\right)y_l\,dx\right]$$

< 1.5. Aerodynamic forces and moments >

❖ Dimensionless coefficients

- The lift and drag coefficients can be obtained in coefficient form :

$$c_l = c_n \cos\alpha - c_a \sin\alpha$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha$$

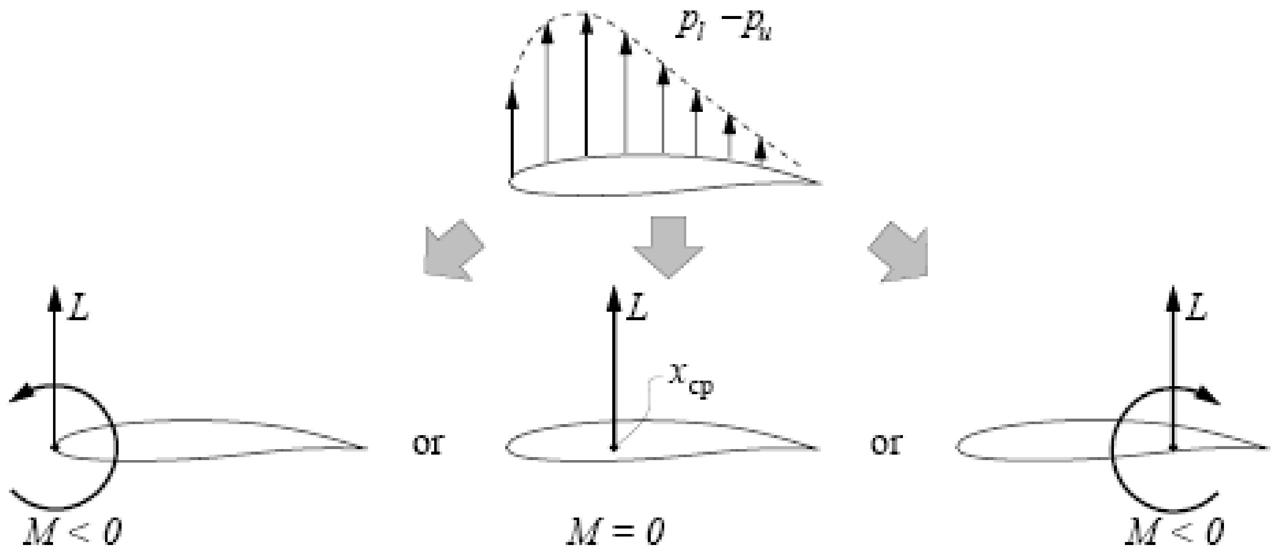
< 1.6. Center of pressure >

The value of the moment M' depends on the reference point. Using the simplified form for the M_{LE} integral, the moment M_{ref} for an arbitrary reference point x_{ref} is

$$M'_{ref} = \int_0^c -\left(p_l - p_u\right)\left(x - x_{ref}\right)dx$$
$$= M'_{LE} + L'x_{ref}$$

< 1.6. Center of pressure >

This can be positive, zero, or negative, depending on where x_{ref} is chosen, as illustrated in the figure.



< 1.6. Center of pressure >

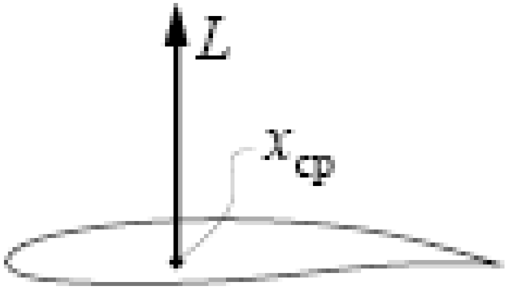
❖ Center of pressure

- Definition : (cont'd)

  At one particular reference location x_{cp}, called the *center of pressure*, the moment is defined to be zero.

$$M'_{cp} = M'_{LE} + L'x_{cp} \equiv 0$$

$$x_{cp} = -M'_{LE} / L'$$

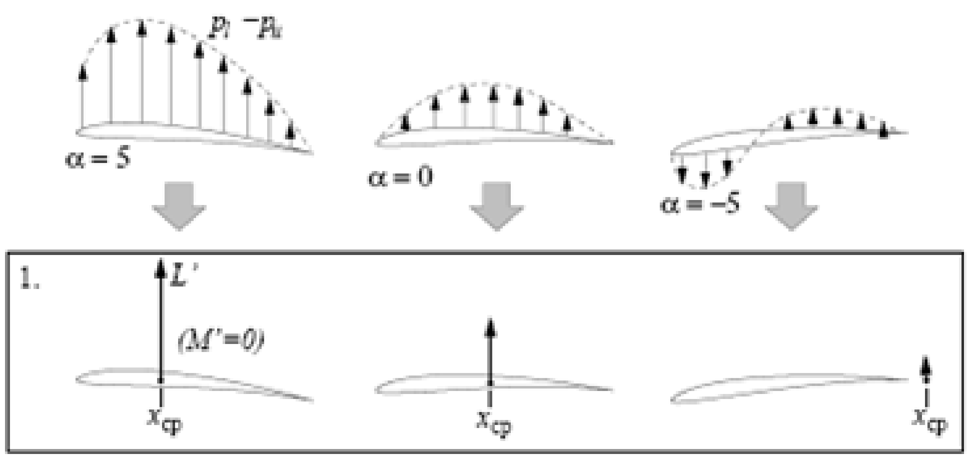


$M = 0$

< 1.6. Center of pressure >

❖ Aerodynamic conventions

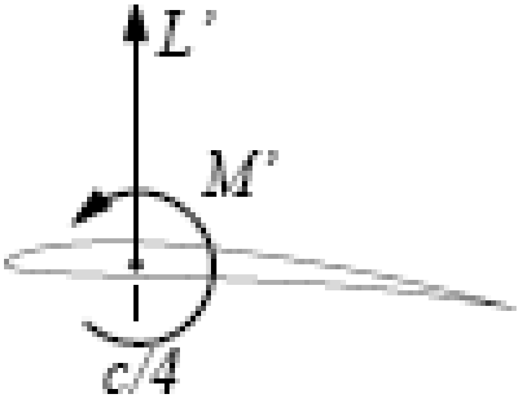
- The figure shows how the L', M', and x_{cp} change with angle of attack for a typical cambered airfoil. Note that with representation 1, the x_{cp} location moves off the airfoil and tends to $+\infty$ as L' approaches zero.



< 1.7. Center of pressure & Aerodynamic Center>

- The center of pressure asymptotes to +∞ or –∞ as the lift tends to zero. This awkward situation can easily occur in practice, so the center of pressure is rarely used in aerodynamics work.

→ quarter-chord point : $x_{ref}=c/4$



< 1.6. Center of pressure & Aerodynamic Center >

❖ Quarter-chord point

- For reasons which will become apparent when airfoil theory is studied, it is advantageous to define the 'standard' location for the moment reference point of an airfoil to be at its quarter-chord location.

  → $x_{ref} = c/4$.

- The corresponding standard moment is usually written without any subscripts.

$$M'_{c/4} \equiv M' = \int_0^c -(p_l - p_u)(x - c/4)dx$$

< 1.6. Center of pressure & Aerodynamic Center >

❖ Aerodynamic conventions

- As implied above, the aerodynamicist has the option of picking any reference point for the moment. The lift and the moment then represent the integrated $p_l - p_u$ distribution. Consider two possible representations:

  - A resultant lift L' acts at the center of pressure $x = x_{cp}$. The moment about this point is zero by definition: $M'_{cp} = 0$. The x_{cp} location moves with angle of attack in a complicated manner.

  - A resultant lift L' acts at the fixed quarter-chord point $x = c/4$. The moment about this point is in general nonzero: $M'_{c/4} \neq 0$.

< 1.6. Aerodynamic Center >

❖ Aerodynamic conventions

- Fixing the moment reference point, as in representation 2, is a simpler and preferable approach. Choosing the quarter-chord location for this is especially attractive, since *M'* then shows little or no dependence on the angle of attack.